Research Statement

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My current research interests lie in analytical and computational aspects of inverse problems for partial differential equations. I focus on practical inverse problems arising in exploration geophysics and medical imaging, especially the parameter identification problems. These problems are inherently interdisciplinary involving collaboration from various disciplines, such as mathematics, physics, chemistry, medicine, biology and engineering, etc. In many cases, these nondestructive evaluation can be formulated as an inverse boundary value problem (or Calderón's problem) or a coupled physics problem (or hybrid imaging). Analyzing and solving the problem require a range of understanding and insights of mathematical techniques from ordinary and partial differential equations, functional analysis, complex analysis, matrix theory and numerical methods. I have gained valuable research experience at Purdue University during my PhD study, at ExxonMobil Research & Engineering Company as a visiting researcher and Institute for Mathematics and its Applications as an Industrial Postdoc. In particular, I have been involved in the following projects:

- Stable determination in Magneto-acoustic tomography with magnetic induction (MAT-MI).
- Effects of parameterization on the parameter estimation problems using Gauss-Newton method.
- Convergence analysis of iterative methods in Banach spaces.
- Conditional Lipschitz stability of inverse boundary value problems.
- Multi-level reconstruction for inverse medium problem with multi-frequency data.

Below I briefly present an overview of my research interests and plan for future research.

Magneto-acoustic tomography with magnetic induction

Electrical conductivity of the biological tissues provides important information for clinical and research purposes, such as cancer detection and functional imaging. MAT-MI is a new coupled-physics modality for electrical impedance imaging integrating magnetism and ultrasound. In experiment, the object of interest is placed in a static magnetic field and a pulsed magnetic field. The induced eddy current in the presence of the static magnetic field results in ultrasonic vibration of the object through the Lorenz force. Such acoustic waves propagate and are then collected by the transducers around the object to reconstruct the conductivity. Mathematically, the underlying problem involves parameter reconstruction from the knowledge of internal functionals. Such inverse problems are often referred to as *hybrid imaging*, or *coupled-physics imaging* or *multi-waves imaging*. The capability of MAT-MI to provide high spatial resolution features of biological tissue has been demonstrated experimentally [23, 14, 22] and makes it an attractive modality. During my postdoctoral fellowship at IMA, we provide a mathematical analysis and numerical methods for MAT-MI.

Typically, a coupled physics imaging is modeled by two inverse problems. MAT-MI is decoupled into two steps. The first step involves an inverse source problem for an acoustic wave equation and has been studied extensively. The second step is to reconstruct the spatially varying electrical conductivity with the knowledge of the internal functional, which is the acoustic source term obtained in the first step. We concentrate on the second step. The propagation of the electromagnetic waves is governed by Maxwell's equations. In the experiment setting of MAT-MI, the magnetic field stimulation is short and the conductivity is assumed invariant with respect to time. Hence, we can separate the time and spatial variables. Furthermore, we eliminate the time dependency and derive a mathematical model for the second step of MAT-MI, which is equivalent to a Neumann problem for elliptic equations of second order. With a multiplier method and simple integration by parts, we prove that, if the conductivity is a priori known near the boundary, then it can be uniquely and stably reconstructed from one internal data. For the numerical scheme, we introduce the linearized problem and examine the Fréchet differentiability of the forward operator. With the upper bound and Lipschitz constant of the Fréchet derivative and the stability constant obtained, a linear convergence rate of the method of steepest descent is expected.

Effects of parameterization on the parameter estimation problems

Inverse problems in seismic exploration are usually characterized by a high computational cost because of the huge amount of data and the complexity of solving wave equation repeatedly. Iterative linearized optimization methods, such as the Gauss-Newton algorithm, are extensively used in parameter estimation problems. It has been noticed by the geophysical community that the inversion can be greatly speeded up by choosing a proper parameterization [6].

We use the idea of nonlinearity to interpret this observation and propose a *parameterization selection method* [18]. Based on a sampling method and regression analysis, we build a framework to measure the quantified nonlinearity and use it to evaluate a parameterization to predict the error-decay of an inversion. In this work, we focus on the influences of the representation of the *inversion parameters*, i.e., a bijection from the modeling parameters to the inversion parameters. This suggestion is designed to take advantage of the existing inversion machinery and require little additional effort to incorporate according to the chain rule.

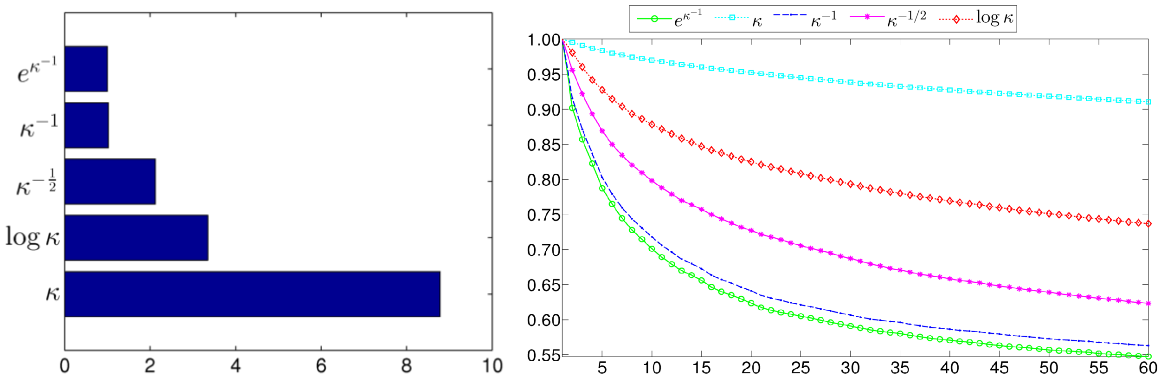
The implementation involves the use of Haar wavelet and Fourier series to represent model and data in low dimensional spaces respectively with possibly minimal error in measuring nonlinearity. The proposed method only requires the forward simulation and is naturally parallelable, hence the computational cost is low. Our solution significantly reduces the number of iterations. In Figure 1 and 2, we show the agreement between the prediction and inversion results for single and multiple parameter reconstruction. The left figures show the quantified nonlinearity for associated inversion parameters in terms of the modeling parameters (κ or κ, μ). The right figures show the relative error-decay, $\frac{\|\kappa_k - \kappa^\dagger\|}{\|\kappa_0 - \kappa^\dagger\|}$, of the parameters for different parameterization, which is always calculated using the modeling parameters.

Iterative methods in Banach spaces

Nonlinear Landweber iteration in Banach spaces

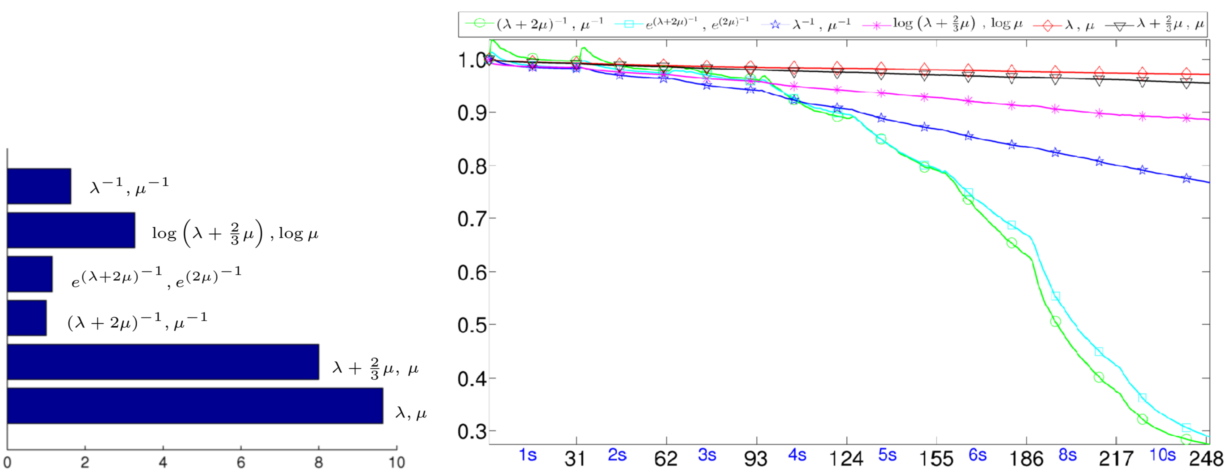
To solve an operator equation $F(x) = y$, the Landweber iteration, which is a gradient method with respect to the functional $x \mapsto \|F(x) - y\|^p$ in the Banach space setting, is widely used. Extensive research has been carried out to study convergence of the Landweber iteration and its modifications, mainly in Hilbert spaces. Schöpfer, Louis & Schuster [20] presented a nonlinear extension of the Landweber method to Banach spaces using duality mappings.

We establish the convergence result and obtain the convergence rate of the nonlinear Landweber iteration in Banach spaces [8]. The primary goal of this work is to replace the usual tangential



(a) Performance prediction [1]   (b) Error-decays (60 iterations)

Figure 1: Agreement between the prediction and inversion results (single parameter)



(a) Performance prediction   (b) Error-decays (248 iterations with time continuation)

Figure 2: Agreement between the prediction and inversion results (multiple parameters)

cone condition and source conditions with a local Hölder(Lipschitz) type stability estimate in the convergence analysis. The motivation here comes from two major factors. First, the usual tangential cone condition and source conditions have been proven to be very difficult to verify for practically relevant inverse problems. Second, the stability, which is a desirable property for any robust algorithm, has been established for many inverse problems and the analysis usually depends on only the forward operator, not the true solution itself. A sublinear convergence rate is derived with Hölder stability provided. And it switches to a linear one if Lipschitz stability is satisfied. We also provide an explicit form of the a priori step size.

A projected steepest descent iteration

In [9], we mainly focus on generalizing our previous results on the Landweber iteration [8] in the following aspects:

[1] shorter bars indicate better performance

- Use an a posteriori step size, instead of an a priori step size.
- Relax the stability condition.
- Include the cases of noisy data.
- Involve regularization procedure.

The steepest descent method proposed here is a generalization of the steepest descent method for unconstrained linear problems (see for example [12]). Our main result concerns restricted convergence of the projected steepest descent iteration with a certain Lipschitz type stability condition on a closed, convex subset of the preimage space. The stability condition here is a relaxed version of the one in [8], in which it is assumed on the whole preimage space.

Note that in general, one can at best expect logarithmic stability in many inverse problems. See, for example, [16, 10]. Motivated by [1, 5], we start to investigate how to improve the logarithmic stability to Lipschitz stability with some additional a priori knowledge on the potentials for the Schrödinger type equation, which will be discussed below. Pursuing this further, for the iterative methods, we consider another question: whether the convergence results hold true for this class of problems. As a consequence, we relax the set, on which we assume the Lipschitz stability, from the whole space to a closed, convex subset. Since the range of the Fréchet derivative at the present iterate may not be within this *stable subset*, it is natural to introduce the projection as a regularization procedure. We use Bregman projection, which bears the non-expansiveness property, according to the Banach space setting.

This result is related to two areas of iterative regularization methods, which are steepest descent algorithms for solving nonlinear inverse problems [17, 19, 13] and projected iteration regularization techniques for the solution of inverse problems with convexity constraints. The latter have been analyzed mostly in the context of *linear* inverse problems (see, for example, [11]) and later as accelerated methods in [7]. Accelerated methods have been modified to nonlinear problems by [21]. The main differences between our work and the above mentioned papers are the conditions under which we prove convergence. In fact, instead of source and nonlinearity conditions (as in [17, 19]), we assume certain conditional Lipschitz stability of the inverse problem. This is a novel view point raised in our previous work on the Landweber iteration [8].

A multi-level scheme

In the convergence analysis of the projected steepest descent iteration [9], the *stable subset* on which we assume Lipschitz stability is fixed and a uniform stability constant is assumed. This stability constant plays an important role in determining the convergence radius and the step size. In a nutshell, the instability of the iterative methods results from the nonlinearity of the problem measured by the stability constant over the *stable subset*.

We propose a multi-level scheme taking into account the possibility that some parameters in the operator, which define the inverse problem, change the data and affect the *stable subsets* and the accuracy of approximations. We combine all known controllable factors to an abstract level number. The nature of our scheme is intimately connected to finding sparse, albeit approximate, representations of the solution. We choose carefully the *stable subsets* so that the growth of the stability constants and the decrease of the approximation errors, which stand for the nonlinearity and accuracy respectively, are moderate. Thus, the outcome of iterations on a coarse level gives a good initial guess on a finer level and we can continue this procedure until the discrepancy principle is satisfied. In this way, our multi-level approach leads to a radius of convergence significantly larger than the one in the single level approach.

Conditional Lipschitz type stability for inverse boundary value problems

In general, one can at best expect a stability estimate with logarithmic type in many inverse problems. For example, Mandache [16] and Di Cristo and Rondi [10] proved by examples that the logarithmic stability estimate is indeed optimal for the inverse conductivity problem and inverse inclusion and scattering problem, respectively. This logarithmic type stability for the Calderón type inverse boundary value problems is rather weak, in the sense that even small changes in the boundary measurements, so-called Dirichlet-to-Neumann (DtN) map, can result in large changes in the reconstructed interior parameters.

Therefore, in order to have better stability estimates, one needs to formulate the problem in some different fashion. Alessandrini and Vessella [1] provided Lipschitz stability estimates for the inverse conductivity problem assuming that the conductivity is piecewise constant with a bounded number of unknown values. Following the same spirit, we investigate the Lipschitz stability estimates for the inverse boundary value problem of the Helmholtz equation [4]. In this context, the stability estimate provides quantitative information on the ill-posedness of the inverse problem and thus, the difficulty on the numerical scheme.

Essentially, we establish a Lipschitz stability estimate

$$\|q_1 - q_2\|_{L^\infty} \leq C\|\Lambda_1 - \Lambda_2\|_{\mathcal{L}(H^{1/2},H^{-1/2})}$$

for potentials q_i and their DtN map Λ_i, $i = 1, 2$, if the potentials are piecewise constants with jumps on a finite number of subdomains. This result is applicable to the complex potentials and partial data cases.

We construct a singular function using two Green's functions of the Schrödinger type equations corresponding to two different potentials. Then, we characterize the rate of blow-up of the singular functions finding lower and upper bounds in terms of the distance of the singularity from the interface of the subdomains. More precisely, on the one hand, with the aid of the quantitative estimates of the *propagation of smallness*, which is proved mainly by the *three spheres inequality*, we can estimate the asymptotic behavior of the singular function from above near the interface of subdomains. On the other hand, by using L^p estimates of the Green's function of the Schrödinger equation and the fundamental solution to the Laplacian operator, we establish the lower bound. These two bounds give us a recursive inequality, by applying which finite times, we obtain the Lipschitz stability estimate.

The main difference from [1, 5] concerns the construction of the singular function, which turns out to be a key ingredient. Especially in dimension 2 and 3, we choose some first order derivative of the Green's function so that the blow-up rate of the singular function is as desired.

Inverse medium problem with multi-frequency data

This project is devoted to the investigation of the application of the multi-level projected steepest iteration [9] to the inverse medium problem with multi-frequency data. The key to the proof is to balance the stability, convergence radius and approximation error as level number increases.

In [9], we use three constants to characterize the forward operator: upper bound and Lipschitz constant of its Fréchet derivative and stability constant. In this application, we first study the dependence of these constants with respect to the frequency and domain partition. We combine the frequency and domain partition to define the level number. The reason is that neither of them alone can afford enough flexibility on the trade-off among stability constant, convergence radius and approximation error. It is also important to note that the stability constant depends on the number of subdomains exponentially [4].

With aid of the asymptotic behaviors with respect to the frequency and domain partition, we prove two related results. First, we show that, for any fixed domain partition and true solution, the convergence radius goes to infinity as the frequency goes to zero. Note that the accuracy limitation depends on the domain partition. Hence the iterations in this level may not give us a high resolution result. Second, we prove that, with a careful selection of the levels, the iteration result of the present level is a good initial point for the next level until the desired accuracy is achieved. The key ingredient here is to compensate the instability caused by pursuing high resolution.

Future research plan

During the time as a PhD student and a postdoctoral researcher, I have established a great network of collaborators in both industry and academia. My future research plan revolves around the ideas of mathematical modeling, stability analysis and reconstruction algorithm for interdisciplinary inverse problems. In addition to the extensions to the projects described above, I am also interested in some new directions. Below I briefly describe several ongoing and potential projects.

Quantitative thermo-acoustic tomography (QTAT)

We are working on relaxing the regularity assumption in the Quantitative thermo-acoustic problems [3]. Our investigation suggests that the uniqueness and stability results can be extended to bounded measurable conductivities using a L^∞ estimate of the complex geometrical optics (CGO) solutions.

Numerical aspect of MAT-MI

The numerical aspect of MAT-MI is of great importance. Aiming to provide a real-time imaging tool, we are working on designing a new reconstruction method involving an adjoint state method.

Effects of the parameterization

There are many directions for future works on the novel *parametrization selection method*. First, even though the detail of the implementation and numerical experiments are based on the inverse medium problem for wave equations, it is clear that this parametrization selection method can be applied to a wide variety of nonlinear inverse problems, especially to the parameter identification problems. Seeking a low-rank compression of the parameter and data spaces will be the leading issue. Second, the present work focuses on the effects of parametrization on the nonlinearity and only algebraic combinations of some basic functions of the parameters are used for testing. The proposed method offers a tool to quantify the effects. For further speeding up the error-decay, deeper investigation of more nontrivial parameterization is needed. Third, to applied the method, we need to have several candidate parametrization. It is of great interest to extend the method to a *parametrization searching method*, which is a method of searching for the optimal parametrization based on a large number of sampling parameter models and their data.

Source conditions and stability estimates

In recent years, a series of source conditions have been developed to carry over convergence rate results for the Tikhonov regularization methods. Under the Hilbert space setting, we propose a new source condition and use it to clarify some assertion on the equivalence of standard and variational source conditions for linear inverse problems [2]. We would like to extend this novel source condition to nonlinear problems and investigate its relation with stability estimates.

Multi-level scheme

The multi-level projected steepest iteration proposed in [9] is a valuable tool in solving nonlinear inverse problems. A natural continuation of this project is to apply it to the reconstruction of the Lamé parameters for the elastic wave equation. The key aspect of the reconstruction is the stability. I am interested in applying this method to a wide range of practical inverse problems.

Lipschitz type stability

This is a follow-up project to the one on the Helmholtz equation. We consider the time harmonic elastic wave equation with isotropic medium. We aim to provide a Lipschitz stability estimate,

$$\|\lambda_1 - \lambda_2\|_{L^\infty} + \|\mu_1 - \mu_2\|_{L^\infty} \leq C\|\Lambda_{\lambda_1,\mu_1} - \Lambda_{\lambda_2,\mu_2}\|_{\mathcal{L}(H^{1/2},H^{-1/2})}$$

for the reconstruction in the case of piecewise constant Lamé parameters. In this case, the regularity results of the solutions to the system are shown in a recent work [15]. With these estimates we anticipate to obtain De Giorgi-Nash type oscillation estimates for the vector case, which lead to the construction of the Green's matrix and certain L^p estimates and pointwise estimates. Then, a singular function can be constructed in analogy with the scalar wave case. The arguments here can be formulated as a framework for the conditional stability analysis for many inverse boundary value problems. I also would like to extend it to Maxwell's equations.

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